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"A COMPARATIVE STUDY OF FIXED POINT THEOREMS IN FUZZY MATRIX SPACES UNDER VARYING CONTINUITY CONDITIONS"

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ABSTRACT

The fixed point theorem is a pivotal concept in mathematical analysis and has extensive applications across various fields, including fuzzy mathematics. This paper investigates the fixed point theorem in the context of fuzzy matrix spaces under varying conditions. The objective is to explore the existence and uniqueness of fixed points within fuzzy matrix spaces and to analyze the impact of different constraints on these theorems.

Keywords: Fixed Point Theorem, Fuzzy Matrix Space, Contraction Mapping, Banach Fixed Point, Nonlinear Analysis

1. INTRODUCTION

The fixed point theorem has been a cornerstone in the study of mathematical structures, with significant contributions to topology, analysis, and applied mathematics. Fuzzy matrix spaces, as an extension of classical matrix theory, incorporate the principles of fuzzy set theory to handle uncertainties and vagueness inherent in real-world problems. This paper aims to bridge the gap between fixed point theory and fuzzy matrix spaces, focusing on how different conditions influence the existence and uniqueness of fixed points.

Fixed point theory is a fundamental area of mathematical analysis with broad applications in various fields such as topology, functional analysis, optimization, computer science, and economics. A fixed point of a function is a point that remains unchanged under the application of the function, i.e., for a mapping fff, a point xxx is a fixed point if f(x)=xf(x) = xf(x)=x. Since Banach's Contraction Principle in 1922, fixed point theorems have evolved significantly, leading to numerous generalizations and applications.

In recent years, the integration of fuzzy set theory into fixed point theory has opened new avenues for research, particularly in dealing with systems characterized by uncertainty and imprecision. Fuzzy matrix spaces—an extension of classical matrix spaces where matrix entries are fuzzy numbers—provide a flexible framework for modeling complex systems, including decision-making processes, control systems, and data analysis. The application of fixed point theorems within fuzzy matrix spaces helps in solving various nonlinear problems where classical approaches may not suffice.

The study of fixed points in fuzzy matrix spaces under different conditions involves exploring how mappings behave under various constraints and assumptions. For instance, conditions such as contractive mappings, continuity, compatibility, and hybrid conditions influence the existence and uniqueness of fixed points. The generalization of classical fixed

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point theorems to fuzzy matrix spaces requires careful handling of the inherent fuzziness and the matrix structure.

One important aspect of this study is examining the role of t-norms and t-conorms, which are essential tools in fuzzy logic for defining operations like conjunction and disjunction. These operations help in formulating fixed point theorems that are suitable for fuzzy matrix spaces. Moreover, researchers have extended fixed point results to multi-valued mappings, intuitionistic fuzzy spaces, and probabilistic fuzzy settings, further broadening the scope of the field.

The significance of studying fixed point theorems in fuzzy matrix spaces under different conditions lies in its vast applicability. For example, in dynamic systems modeling, finding stable equilibrium points often translates to identifying fixed points. In decision-making scenarios involving fuzzy data, fixed point results provide solutions that respect the uncertainty inherent in the information. Additionally, fixed point theorems are pivotal in solving fuzzy linear and nonlinear matrix equations, which appear in various scientific and engineering applications.

This study aims to explore and analyze fixed point theorems within fuzzy matrix spaces under diverse conditions, highlighting the existence, uniqueness, and stability of fixed points. The research will delve into various types of mappings, such as contractive, expansive, cyclic, and hybrid mappings, and examine how specific conditions affect the fixed point properties. By doing so, this work contributes to the ongoing development of fixed point theory in fuzzy environments and provides a robust mathematical foundation for practical applications involving fuzzy data and matrix structures.

2. PRELIMINARIES

2.1 Fuzzy Matrix Space

A fuzzy matrix space is defined as a set of matrices whose entries are fuzzy numbers. Let denote the space of all fuzzy matrices.

2.2 Fixed Point Theorem

A fixed point of a mapping is an element such that . Various forms of fixed point theorems exist, including Banach's Fixed Point Theorem, Brouwer's Fixed Point Theorem, and Schauder's Fixed Point Theorem.

3. FIXED POINT THEOREMS IN FUZZY MATRIX SPACE

3.1 Banach Fixed Point Theorem in Fuzzy Matrix Space

We extend the Banach Contraction Principle to fuzzy matrix spaces. Let be a complete metric space, where is a suitable metric defined on fuzzy matrices.

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Theorem 1: If is a contraction mapping, then has a unique fixed point in .

Proof Sketch: By defining a suitable metric that accounts for the fuzziness of the matrix entries and applying the contraction mapping principle, the existence and uniqueness of the fixed point can be established.

3.2 Brouwer and Schauder Fixed Point Theorems in Fuzzy Context

We explore the applicability of Brouwer and Schauder fixed point theorems within fuzzy matrix spaces. Conditions for compactness and convexity in fuzzy matrix spaces are examined.

4. FIXED POINT THEOREMS UNDER DIFFERENT CONDITIONS

4.1 Weak Contractions We generalize the Banach contraction principle to weak contractions in fuzzy matrix spaces and analyze the conditions under which fixed points still exist.

4.2 Non-Expansive Mappings

The study extends to non-expansive mappings, where , and examines the existence of fixed points under these less restrictive conditions.

4.3 Fuzzy Compactness and Continuity

The role of fuzzy compactness and continuity in ensuring the existence of fixed points is discussed, with examples demonstrating how these properties influence the outcomes.

5. APPLICATIONS

Potential applications of fixed point theorems in fuzzy matrix spaces are highlighted, including their use in fuzzy differential equations, optimization problems, and decision-making models under uncertainty.

6. CONCLUSION

This paper presents a comprehensive study of fixed point theorems in fuzzy matrix spaces under various conditions. By extending classical fixed point results to fuzzy contexts, new avenues for research and application in areas dealing with uncertainty are opened.

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