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SOME NEW RESULTS ON BLITACT GRAPH OF A GRAPH

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ABSTRACT

In this paper, we obtained the number of lines in the blitact graph, when G is a path, Blitact graph of a path to be (p-4)-minimally nonouterplanar, when P≥5, For any connected graph G, Blitact graph $B_m(G)$ is not maximal minimally nonouterplanar and If G is a triangle together with a path of length k adjoined to some point, then Blitact graph $B_m(G)$ is k-minimally nonouterplanar.

3.1 INTRODUCTION

In this section, the concept of blict graph B_n (G) and blitact graph B_m (G) of a graph G are given, introduced by Kulli and Biradar [2].

If B = (u_1, u_2,..u_r; $r \geq 2$) is a block of G, then we say that point u_1 and block B are 'incident' with each other, as are u_2, and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut point, then they are 'adjacent' blocks. The blocks, cut points and lines of a graph are called its 'members'.

The 'Blict graph' $B_n(G)$ of a graph G is the graph whose set of points is the union of the set of blocks, cut points and lines in which two points are adjacent if only if the corresponding blocks and lines of G are adjacent or the corresponding members are incident.

The 'Blict graph' $B_m(G)$ of a graph G is the graph whose point set is the union of the set of blocks, cut points and lines of G and in which two points are adjacent if and only if the corresponding members of G are adjacent or incident. In Figure 1, a graph G and its blict graph B_n (G) and blitact graph $B_m(G)$ are shown.

In 1975, Kulli[1] introduced the idea of minimally nonouterplanar graph.

The innerpoint number i(G) of a planar graph G is the minimum possible number of point not belonging to the boundary of the exterior region in any embedding of G in the plane. Obviously, G is outerplanar if and only if i(G) =0.

A graph is said to be k-minimally nonouterplanar if i(G)=k, k≥1.

A 1-minimal nounouterplanar graph is maximal minimally nonouterplanar if no line can be added without losing minimally nonouterplanarity.

The following will be useful in the proof of our results.

Remark 1.[2]

If G = K1,p, P≥ 2 then $B_n(G)$ (or $B_m(G)$ = K_{p+1}, K_{p+1}

Theorem A : [2]

A graph is planar if and only if it has no subgraph homomorphic to kor k_3.

Theorem B:[2]

The blict graph $B_n(G)$ of a graph G is planar if and only if G satisfies the following

1. G is planar
2. The degree of each point of G is at most three.
3. A cutpoint is not adjacent to other three cutpoints.
4. A cutpoint incident with a nonline block B is not adjacent to other two cut points either of one is not incident with B.

If a block B ha two non-adjacent cutpoints then either of one should not be adjacent to other cutpoint which is not incident with B.

Theorem C:[2]

The blict graph $B_n(G)$ of a graph G is minimally nounouterplanar if and only if G satisfies following condition.

1. Deg v ≤ 3 for every point v of G and
2. G is a block with exactly two point of degree 3 and these are adjacent.
3. G is a cycle together with an end line adjoined to some point or
4. G is a path of length 4

Theorem D:[2]

Let G be a connected (p,q) graph, then $B_n(G)=L(G) \cup K_1$ if and only if G is a block.

Theorem E:[2]

The blitact graph $B_m(G)$ of a graph G is minimally nonouterplanar if and only if G satisfies the following conditions.

1. Degv ≤ 3 for every point v of G and
2. G is a block with exactly two points of degree 3 and these are adjacent . Or
3. G is a cycle together with an end line adjoined to some point or
4. G is a path of length 4.

Theorem F:[2] Let G be a connected (p,q) graph, then $B_m(G) = L(G) \cup K_1$ if and only if G is a block.

Theorem G:[3] If G is a path with p ≥5 points then the blict graph B_n (G) (p-4) minimally nonouterplanar.

Main Results

2. SOME RESULTS ON BLITACT GRAPH OF A GRAPH

In the following theorem we obtain the result which determines the number of lines in the blitact graph when G is a path.

Theorem.1

If G is a path of length n, (n ≥ 2)

Then $|E(B_n(G))|=6(n-1)$ and

$|E(B_m(G))|=6(n-1) + (n-2)$

Proof:

We prove the result by induction on n suppose n=2, then G = $K_{1,2,}$ by Remark 1, $B_n(G)$ (or $B_m(G)=K_3$ K_3, which has 6 liens.

Hence the result is true for n=2

Suppose the result is true for n=K-1

Assume G is a path of length K.

Let $e_j=(v_j,v)$ be an endline of G, delete from G the line e_j. Then the resulting graph G is a path of length K-1.

By inductive hypothesis.

$|E(B_n(G))|=6(K-1)$ and

$|E(B_m(G))|=6(K-2) + (k-3)$

Now again join the line $e_j=(v_j,v)$ to an end line $e_i=(v_i,v_j)$ of G_1 resulting the graph G. The graph $B_n(G)$ is obtained form $B_n(G_1)$ with additional point ej, b_j and v_j, where b_j is an end block incident with the cut point v_j since the line and block coincide in a path. By the definition of the blict graph, the adjacency of additional points with the points of $B_n(G)$ is such that e_j is joined to e_i and both are joined the v_j. In this process 6 new lines are added to $B_n(G_1)$.

The blitact graph $B_m(G)$ is obtained from blict graph $B_n(G)$ by joining the points which are corresponding to the adjacent cut points of G. It is known that the number of cutpoints in a path of length k is equal to (k-1).

Therefore $|E(_n(G))|6(k-2)+6=6(k-1)$

The blitact graph $B_m(G)$ is obtained from blict graph $B_n(G)$ by joining the point which are corresponding to the adjacent cut points of G. It is known that the number of cutpoints in a path of length k is equal to (k-1).

The adjacency of points in Bm(G) corresponding to these (k-1) consecutive adjacent cutpoints produce (k-2) lines.

Therefore $|E(B_m(G))|=E\ (B_m(G))\ | + k-2$

$|E(B_m(G))|=6\ (k-1) + (k-2)$

Hence the result is true for all values of n, n ≥ 2. This completes the proof of the theorem.

We now deduce the necessary condition for blict graph of a path to be (p-4) – minimally nonouterplanar, when p ≥ 5.

Theorem.2

If G is a path with p≥5 points then the blitact graph $B_m(G)$ is (p-4)-minimally nonouterplanar.

Proof:

Suppose G is a path with p≥5 points. The blitact graph $B_m(G)$ is obtained from the blict graph $B_n(G)$ by joining the points which are corresponding to the adjacent cutpoints of G and the adjacency of such points does not alter the minimally nonouterplanarity of $B_n(G)$. That is the number of innerpoints in B_m (G) is equal to that of $B_n(G)$ and hence by theorem G, (If G is a path with p ≥5 points then the blict graph $B_n(G)$(p-4) minimally nonouterplanar. $B_m(G)$ is also (p-4) minimally nonouterplanar.

In the following theorem, we obtain a characterization of graphs whose blitact graphs are maximal minimally nonouterplanar.

Theorem. 3

The blitact graph $B_m(G)$ of a graph G is maximal minimally nonouterplanar if and only if G is a path of length four.

Proof:

Suppose $B_m(G)$ is maximal minimally nonouteprlanar. Clearly $B_m(G)$ is minimally nonouterplanar. Then G satisfies the conditions as stated in Theorem E.

Suppose Δ (G) $\leq$ 3. Now we consider the following cases.

Case 1:

Suppose G is a block with exactly two points of degree three and these are adjacent. Then by theorem F (let be a connected (p.q) graph, then $B_m(G)$= L(G) UK_1 if and only if G is a block). $B_m(G)$=L(G) UK_1 and by joining the isolated point with some point which is on the exterior region of the subgraph L(G) does not alter the minimally nonouterplanarity of $B_m(G)$, so Bm(G) is not maximal minimally nonouterplanar.

Case 2:

Suppose G is a cycle together with an endline adjoined to some point. Then the corresponding blitact graph $B_m(G)$ has two nonadjacent points (see the points e_i and B_i of Fig.2) whose join does not alter the minimally nouterplanarity of $B_m(G)$, and again $B_m(G)$ is not maximal minimally nonouterplanar. Thus G is a path of length 4.

Conversely suppose G is a path of length 4. Then by theorem C, Bm(G) is minimally nonouterplanar and is so connected that it is not possible to add a line without losing minimally nonouterplanarity.

Thus $B_m(G)$ is maximal minimally nonouterplanar.

This completes the proof of the theorem.

Theorem. 4

If G is a triangle together with a path of length k adjoined to some point, then $B_m(G)$ is k-minimally nonouterplanar.

Proof:

Suppose G is a triangle together with a path of length one adjointed to some point. Then by theorem E, Bm(G) is minimally nonouterplanar. When the length of the path is extended by k(k>1) which is adjoined to some point of the triangle, then the proof of the theorem will be similar to that of theorem.2, (if G is a path with $p \geq 5$ points then the blitact graph $B_m(G)$ is (p-4) – minimally nonouterplanar) and hence we omit the proof.

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