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STUDY OF INTUITIONISTIC FUZZY TOPOLOGY SPACES AND COMPACTNESS AND SEPARATION AXIOMS

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ABSTRACT: In the last few years various concepts in fuzzy sets were extended to intuitionistic fuzzy sets. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics. The term Fuzzy Set (FS), as a formalization of vagueness and partial truth, and represents a degree of membership for each member of the universe of discourse to a subset of it. This also provides a natural frame work for generalizing many branches of mathematics such as fuzzy rings, fuzzy vector spaces and fuzzy topology. The present paper consisting of study which is devoted to the study of Intuitionistic fuzzy topological spaces and compactness and separation axioms. After giving the fundamental definitions we have discussed the concepts of intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, and separation axioms, that is, intuitionistic fuzzy regular space.

KEY WORDS: Fuzzy Set (FS), Intuitionistic fuzzy topological spaces, compactness and separation axioms, fuzzy continuity.

I.INTRODUCTION

Ever since the invention of topological spaces, many researchers have been paying remarkable contribution in this field. By investigating different properties on classical topological spaces, they also added new notions for its generalization. The fundamental concept of a fuzzy set was introduced by Zadeh in 1965 and fuzzy topology by Chang in 1968 [1]. As generalizations of fuzzy set, the concept of intuitionistic fuzzy set was introduced by Atanassov which take into account both the degrees of membership and non-membership subject to the condition that their sum does not exceed.

Primarily in the area of set theory, fuzzy mathematics differs from conventional mathematics. Fuzzy mathematics had been introduced just few years ago, it is full of topics. It is used widely in many sectors such as, vehicles, traffic system where logic circuit controls anti-skid brakes, transmissions, and other operations. We have discussed in this paper about a set, which is more specified than crisp set. It can take a decision between yes or no, i.e. 1 or 0 [2].

In this real physical world, it would be better if we are able to study the objects in a classified way. But most often than not, it cannot be done, because the objects do not exist a specifically definite criterion of association [3]. Many illustrations may be written, the group of animals obviously consists of birds, cats, deer, etc. On the other hand, the things like bacteria, virus, starfish, jellyfish, etc., have an uncertain category regarding the animal's class. The similar type of uncertainty arises for a numerical value like 10 regarding to the "class" of the set of all real values which become higher than 1.

Obviously, "the group of the set of all real values which become higher than 1" otherwise "the class of beautiful women" otherwise "the class of tall men," cannot

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represent sets otherwise groups in the common mathematical logic of these languages [4]. However, the truth continues that such inaccurately called "classes" take part in a vital position in human being thoughts, mainly for the area of prototype abstraction, informative communication and recognition. Research based on the fuzzy sets hypothesis is increasing gradually from the time at the beginning of the hypothesis in mid – 1960S. Now, the concepts and outcomes containing the hypothesis of fuzzy set are relatively remarkable. In addition, various applications based research has been conducted very vigorously and has found more extraordinary outcomes [5].

II.PRELIMINARIES

Fuzzy sets were introduced by Zadeh in 1965 as follows: a fuzzy set A in a nonempty set X is a mapping from X to the unit interval [0, 1], and A(x) is interpreted as the degree of membership of x in A. Intuitionistic fuzzy sets can be viewed as a generalization of fuzzy sets that may better model imperfect information which is in any conscious decision making. Intuitionistic fuzzy sets take into account both the degrees of membership and of non-membership subject to the condition that their sum does not exceed 1.

Let E be the set of all countries with elective governments. Assume that we know for every country $x \in E$ the percentage of the electorate that has voted for the corresponding government. Denote it by M(x) and let $\mu(x) = M(x)/100$ (degree of membership, validity, etc.).

Let $\nu(x) = 1 - \mu(x)$.

This number corresponds to the part of electorate who has not voted for the government. By fuzzy set theory alone we cannot consider this value in more detail. However, if we define $\nu(x)$ (degree of non-membership, non-validity, etc.) as the number of votes given to parties or persons outside the government, then we can show the part of electorate who have not voted at all or who have given bad voting-paper and the corresponding number will be $\pi(x) = 1 - \mu(x) - \nu(x)$ (degree of indeterminacy, uncertainty, etc.). Thus we can construct the set {hx, μ(x), ν(x)i : x ∈ E}. Intuitionistic fuzzy sets (IFS) are applied in different areas.

Basic Operations on IFS

Definition 2.1: Let X be a non empty set, and the IFSs A and B be in the form A = {hx, μA(x), γA(x)i : x ∈ X} and B = {hx, μB(x), γB(x)i : x ∈ X}

1. A ⊆ B iff μA(x) ≤ μB(x) and γA(x) ≥ γB(x) for all x ∈ X.

2. A = B iff A ⊆ B and B ⊆ A.

3. A = {hx, γA(x), μA(x)i : x ∈ X}.

4. A T B = {hx, μA(x) V μB(x), γA(x) W γB(x)i : x ∈ X}

5. A S B = {hx, μA(x) W μB(x), γA(x) V γB(x)i : x ∈ X}

6. [ ]A = {hx, μA(x), 1 − μA(x)i : x ∈ X} 7. h iA = {hx, 1 − γA(x), γA(x)i : x ∈ X}

Definition 1.2.2: The IFS 0∼ and 1∼ in X are defined as

0∼ = {hx, 0, 1i: x ∈ X}

1∼ = {hx, 1, 0i: x ∈ X},

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Where 1 and 0 represent the constant maps sending every element of X to 1 and 0, respectively.

III INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

Definition 3.1: An intuitionistic fuzzy topology (IFT) on a nonempty set X is a family τ of IFS in X satisfying the following axioms

(T1) 0∼, 1∼ ∈ τ

(T2) G1 T G2 ∈ τ , for any G1, G2 ∈ τ

(T3) S Gi ∈ τ , for any arbitrary family {Gi : Gi ∈ τ, i ∈ I}.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space and any IFS in τ is known as intuitionistic fuzzy open set in X.

Proposition 3.1: Let (X, τ) be an IFTS on X . Then we can also construct several IFT on X in the following way

(a) τ0,1 = {[ ]G : G ∈ τ}

(b) τ0,2 = {h iG : G ∈ τ} .

Proof: (a) (T1) 0∼, 1∼ ∈ τ0,1 is obvious.

(T2) Let [ ]G1, [ ]G2 ∈ τ0,1.

Since G1, G2 ∈ τ , therefore G1 T G2 = hx, µ1 V µ2, γ1 W γ2i ∈ τ .

Definition 3.2: Let (X, τ1) ,(X, τ2) be two IFTSs on X. Then τ1 is said to be contained in τ2 if G ∈ τ2 for each G ∈ τ1 . In this case, we also say that τ1 is coarser than τ2.

Proof: Let {τi : i ∈ J} be a family of IFTS on X. We have to show that ∩τi , i ∈ J is an IFT on X .
(i) 0∼ ∈ τi , for every i ∈ J. From this it follows that 0∼ ∈ ∩τi . Similarly, 1∼ ∈ ∩τi

(ii) Let G1, G2 ∈ ∩τi . Then G1, G2 ∈ τi , for every i ∈ J and hence, G1 ∩ G2 ∈ τi , ∀i ∈ J. Thus, G1 ∩ G2 ∈ ∩τi .

(iii) Let {Gj : j ∈ K} ⊆ ∩τi . Then {Gj : j ∈ K} ⊆ τi , for every i ∈ J and hence, S j∈K Gj ∈ τi , ∀i ∈ J. Thus, S j∈K Gj ∈ ∩τi .

Clearly, it is the coarsest topology on X containing all τ 0 i s. Since if τ 0 is any other IFT on X which contains every τi , then obviously it will also contain ∩τi .

IV. COMPACTNESS AND SEPARATION AXIOMS

4.1 Intuitionistic Fuzzy Compactness

Definition 4.1: Let (X, τ) be an IFTS.

(a) If a family {hx, µGi , γGi i : i ∈ J} of IFOS in X satisfy the condition S {hx, µGi , γGi i : i ∈ J} = 1∼ then it is called a fuzzy open cover of X. A finite subfamily of fuzzy open cover {hx, µGi , γGi i : i ∈ J} of X, which is also a fuzzy open cover of X is called a finite subcover of {hx, µGi , γGi i : i ∈ J}.

(b) A family {hx, µKi , γKi i : i ∈ J} of IFCSs in X satisfies the finite intersection property iff every finite subfamily {hx, µKi , γKi i : i = 1, 2, · · · , n} of the family satisfies the condition Tn i=1{hx, µKi , γKi i} 6= 0∼.

Definition 4.2: An IFTS (X, τ) is called fuzzy compact if every fuzzy open cover of X has a finite sub cover.

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Proposition 4.1: Let (X, τ) be an IFTS on X. Then (X, τ) is fuzzy compact iff the IFTS (X, τ0,1) is fuzzy compact.

Proof: Let (X, τ) be fuzzy compact and consider a fuzzy open cover {[ ]Gj : j ∈ K} of X in (X, τ0,1). Since S ([ ]Gj) = 1∼ we obtain W µG = 1, and hence, by γGj ≤ 1 − µGj ⇒ V γGj ≤ 1 − W µGj = 1 − 1 = 0 ⇒ V γGj = 0, we deduce S Gj = 1∼. Since (X, τ) is fuzzy compact there exist G1, G2, · · · Gn such that Sn i=1 Gi = 1∼ from which we obtain Wn i=1 µGi = 1 and Vn i=1(1 − µGi) = 0, that is, (X, τ0,1) is fuzzy compact.

Suppose that (X, τ0,1) is fuzzy compact and consider a fuzzy open cover Gj : j ∈ K of X in (X, τ). Since S Gj = 1∼, we obtain W µGj = 1 and V (1 − µGj) = 0. Since (X, τ0,1) is fuzzy compact there exist G1, G2, · · · Gn such that Sn i=1([ ]Gi) = 1∼, that is, Wn i=1 µGi = 1 and Vn i=1(1−µGi) = 0. Hence µGi ≤ 1−γGi ⇒ 1 = Wn i=1 µGi ≤ 1− Vn i=1 γGi ⇒ Vn i=1 γGi = 0. Hence Sn i=1 Gi = 1∼. Therefore (X, τ) is fuzzy compact.

4.2 Intuitionistic Fuzzy Regular Spaces

Definition 4.3: An IFTS (X, τ) will be called regular if for each IFP p x (α,β) and each IFCS C such that p x (α,β) ∩ C = 0∼ there exists IFOS M and N such that p x (α,β) ∈ M and C ⊆ N. Note: For the simplification of the notation we will write the IFP p x (α,β) as x(α,β) .

Proposition 4.2: If a space X is a regular space then for any open set U and intuitionistic fuzzy point x(α,β) such that x(α,β) ∩ U 0 = 0∼, ∃ an open set V such that x(α,β) ∈ V ⊆ V ⊆ U.

Proof: Suppose that X is a IFRS. Let U be an IFOS of X such that x(α,β) ∩ U 0 = 0∼ and U = hy, µU , γU i. Then U 0 = hy, νU , µU i is an IFCS in X. Since X is regular, therefore ∃ two IFOSs V and W such that x(α,β) ∈ V , U 0 ⊆ W and V ∩ W = 0∼. Now, W0 is an IFCS of X such that V ⊆ W0 ⊆ U. Thus, x(α,β) ∈ V ⊆ V and V ⊆ W0 ⊆ U, so V ⊆ U. Hence, x ∈ V ⊆ V ⊆ U.

Proposition 4.3: Every subspace of regular space is also regular.

Proof: Let X be a IFRS and Y is a subspace of X. To prove that Y is regular. We know that τY = {GY = hx, µG|Y , νG|Y i : x ∈ Y, G ∈ τ}, where G = hx, µG, νGi. Let x(α,β) be an IFP in Y and FY is an IFCS of Y such that x(α,β)∩FY = 0∼. Since Y is a subspace of X, so x(α,β) ∈ X and there exists an IFCS F in X such that the closed set generated by it for Y is FY . Since X is regular space and x(α,β) ∩ F = 0∼, there exist two IFOSs M and N such that x(α,β) ∈ M = hx, µM, νMi and F ⊆ N = hx, µN , νN i. Thus MY = hx, µM|Y , νM|Y i, and NY = hx, µN|Y , νN|Y i are open sets in Y such that x(α,β) ∈ MY and FY ⊆ NY . Hence, Y is a regular subspace of X.

V. CONCLUSION

The purpose of this paper is to introduce and study the compactness in intuitionistic fuzzy topological spaces. Here it define two new notions of intuitionistic fuzzy compactness in intuitionistic fuzzy topological space and find their relation. Also it finds the relationship between intuitionistic general compactness and intuitionistic fuzzy compactness. Here it sees that our notions satisfy hereditary and productive property. Finally this observe that our notions preserve under one-one, onto and continuous mapping.

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