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ADAPTIVE ONE-STEP METHOD FOR ORDINARY DIFFERENTIAL EQUATION SOLVING: IMPROVING EFFICIENCY AND STABILITY

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Abstract:

Ordinary differential equation (ODE) one-step techniques are essential to computational mathematics because they allow for precise and effective numerical approximations of intricate dynamic systems. In this research, adaptive step-size selection and improved stability analysis are used to investigate a novel approach to one-step procedures. By maximising error control and computing efficiency, the suggested approach outperforms conventional explicit and implicit approaches. The benefits of this approach, especially in the solution of stiff and non-stiff differential equations, are shown by theoretical analysis and numerical experimentation.

Keywords: stability, numerical techniques, ordinary differential equations, one-step procedures, and step-size adaptation.

Introduction:

Physics, biology, and economics are just a few of the scientific and technical fields where ordinary differential equations (ODEs) are used. Due to the unavailability of closed-form solutions for many real-world issues, numerical techniques are required for approximation. One-step approaches, including Euler's method and Runge-Kutta methods, offer a compromise between accuracy and processing economy among the many other techniques available.

By presenting an adaptive framework that dynamically modifies step size to increase accuracy while preserving stability, this study seeks to improve on current one-step methods. For stiff ODEs, which present difficulties in numerical calculations because of their quickly fluctuating solutions, the novel method is very helpful.

Objectives:

This study's primary goals are to:

- Examine the drawbacks of traditional one-step approaches to ODE solution.
- To create a better numerical technique that increases stability and accuracy.

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• To improve error control by putting in place an adaptive step-size selection system.
• To use numerical simulations to compare the suggested approach with current methodologies.

One-Step methods: A Synopsis

Multi-step dependencies are avoided by one-step approaches, which use only the present point to calculate the next solution point. These techniques fall into one of two categories: explicit or implicit.

• **Explicit Methods:** Euler's approach and the explicit Runge-Kutta methods use the current point's known slope to estimate the subsequent step. Although they can be unstable for stiff equations, they are computationally cheap.

• **Implicit procedures:** Backward Euler and implicit Runge-Kutta procedures are more stable and appropriate for stiff situations, but they need solving nonlinear equations at each stage. Even though they are widely used, classic one-step approaches have trouble striking a balance between computing cost, accuracy, and stability.

Suggested Approach

1. The Flexible Step-Size Approach:

The suggested approach uses an adaptive mechanism that dynamically modifies the step size depending on error estimations rather than a fixed step size. By taking bigger steps in areas that develop slowly and smaller ones in areas that change quickly, this strategy increases efficiency.

2.Stability Improvement

The approach is improved to handle stiff equations by adding a new stability criteria. A weighted blend of explicit and implicit formulations is included in the suggested improvement, increasing stability without placing an undue computing strain on the system.

3. Algorithm Development

The enhanced one-step method is constructed as follows:

- By estimating the local truncation error,
- adjusting the step size according to a predetermined tolerance,
- applying an implicit correction step in the event that stability conditions are not met
- moving on to the next step using the updated value.

hybrid approach guarantees equilibrium between accuracy and efficiency while preserving stability across various problem domains.

Example: Examine the initial value problem to demonstrate the efficacy of the suggested approach:

$$\frac{dy}{dx} = -2y + x, \quad y(0) = 1$$

Step 1: Explicit Euler Predictor Step

Applying Euler's technique with a starting step size of h: The formula
$y_{n+1}^{(pred)} = y_n + hf(x_n, y_n)$

At $x_0 = 0,\ y_0 = 1$:

$y_1^{(pred)} = 1 + 0.1(-2(1) + 0) = 1 - 0.2 = 0.8$, for h=0.1

Step 2: Estimating Errors

$$E = |y_{n+1}^{(pred)} - y_{true}|$$

is the estimated value for the local truncation error. where the precise answer is denoted y_{true} . h is decreased if E above the tolerance.

Step 3: Backward Euler Integral Correction:

The implicit corrective step is implemented if instability is found:

$$y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$$

We get a more precise approximation by solving for y_1 repeatedly.

Results and Numerical Experiments :

Several benchmark ODEs are used to test the suggested approach, including:
1) A common test case for evaluating numerical stability is the linear test equation.
2)Van der Pol Equation: A nonlinear ODE that illustrates how well the adaptive step-size method works.
3)Lorenz System: A chaotic system that evaluates the method's resilience under challenging situations. According to the results, the novel technique performs better in terms of computing efficiency and error reduction than conventional one-step procedures.

4)The algorithm's adaptability minimises pointless computations without sacrificing accuracy.

Discussion:

According to the results, one-step approaches perform noticeably better when adaptive step-size control and stability enhancements are combined. The suggested method is a flexible tool for numerical calculations as it offers a workable solution for both stiff and non-stiff issues. Additional optimisations, including parallel implementations for high-performance computing applications, can be investigated in future studies.

Limitations:

Although the suggested approach appears promising, there are several drawbacks:
• The extra calculations needed for step-size adaption might make implementation more difficult.
• Additional stability criterion improvement could be required for strongly oscillatory issues.

• The choice of error tolerance determines the method's efficacy, necessitating problem-specific adjustment.

Conclusion :

With adaptive step-size selection and stability improvements, this study presents an enhanced one-step method for solving ODEs. Numerical experiments confirm its superiority over conventional methods, providing a balance between accuracy and computational efficiency. The method can be used in a variety of scientific and engineering fields, opening the door for further developments in numerical analysis.

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