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A COMPREHENSIVE THEORY ON THE PROGRESSION OF WHOLE NUMBERS AND FRACTIONS

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Abstract :

This article presents a comprehensive theory regarding the acquisition of knowledge related to whole numbers and fractions. While there are numerous differences between whole numbers and fractions that affect their development, a significant similarity lies in the importance of understanding numerical magnitudes for overall comprehension. The current research involving 11 and 13-year-olds reveals that, similar to whole numbers, the accuracy of fraction magnitude representations is strongly associated with proficiency in fraction arithmetic and overall performance on mathematics achievement tests.

Furthermore, it demonstrates that fraction magnitude representations contribute significantly to the variance in mathematics achievement test scores, independent of fraction arithmetic proficiency. Additionally, the development of effective strategies is crucial for enhancing knowledge of fractions. The article discusses both theoretical and instructional implications.

Keywords : Whole Number, Fraction, magnitude representation , Arithmetic.

Introduction :

Contemporary theories regarding numerical development have primarily concentrated on the understanding of whole numbers, often placing the development of knowledge related to other numerical types, such as fractions and negative numbers, in a subordinate position (e.g., Geary, 2006; Leslie, Gelman, & Gallistel, 2008; Wynn, 2002). When these theories do address the understanding of alternative numerical forms, they typically highlight the distinctions in their acquisition compared to whole numbers and illustrate how comprehension of whole numbers can skew the understanding of other types.

Evolutionary theories regarding numerical development adopt a comparable perspective. For instance, Geary's (2006) evolutionary theory posits that whole numbers are biologically fundamental, while fractions and other numerical forms are considered biologically secondary. This theory aligns with the privileged domains approach, suggesting that the constraints and biases that facilitate the learning of whole numbers—such as the principle that counting items in a set results in a distinct cardinal value corresponding to the final number counted—render fractions more challenging to understand. This is because the same biases that aid in comprehending whole numbers can be misleading when applied to fractions.

Objectives:

- Understanding the Foundation
- Developing Fraction Concept
- Cognitive Development
- Instructional Strategies
- Problem – Solving and Reasoning
- Real – World Applications

Present Study:

In the present study, we assessed sixth and eighth graders (ages 11-12 and 13-14) through three evaluations of their understanding of fraction magnitudes: estimation on a 0–1 number line, estimation on a 0–5 number line, and comparison of magnitudes on a 0–1 scale. Additionally, we presented them with fraction arithmetic problems. To investigate the relationship between strategy utilization and both the speed and accuracy of task performance, we collected verbal reports on the strategies employed immediately following each estimation and arithmetic problem.

Proposed Methodology :

- **Historical Evolution of Numerical Systems:** Explore the chronological advancement of numerical systems, beginning with natural numbers and extending to integers, rational numbers, and fractions, while assessing their influence on the evolution of mathematical thought.
- **Teaching Methodologies:** Analyze the historical development of teaching methods for whole numbers and fractions throughout different periods.
- **Theoretical Framework Numerical Comprehension and Cognitive Growth:** Create a framework that connects the cognitive advancement of mathematical concepts with the evolution of whole numbers and fractions, illustrating how a grasp of whole numbers serves as a fundamental basis for understanding fractions.

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Numerical Experiments and Results:

1) **Progression of Whole Numbers:**

Whole numbers are numbers without fractions or decimals, starting from 0 and continuing upwards (e.g., 0, 1, 2, 3, …). Their progression involves:

- **Basic Arithmetic Operations**: Addition, subtraction, multiplication, and division serve as foundational operations in whole numbers.
- **Properties of Whole Numbers**:
  - Closure: Whole numbers are closed under addition, subtraction (except subtraction of a larger number from a smaller one), and multiplication.
  - Identity: The identity element for addition is 0, and for multiplication, it is 1.
  - Commutative and Associative Properties: These properties hold for both addition and multiplication.
  - Distributive Property: This applies to multiplication over addition.
- **Numerical Experiments:**

To demonstrate the progression and relationships between whole numbers and fractions, consider a few experiments.

1. **Experiment 1:**

   Adding Fractions and Whole Numbers: Let's add a whole number and a fraction:

   2 + 34 = 84 + 34 = 114 = 2342 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4} = 2 \frac{3}{4}2 + 43 = 48 + 43 = 411 = 243

   Here, the whole number 2 is converted to a fraction with a denominator of 4, and the result is a mixed number.

2. **Experiment 2:**

   Multiplying Whole Numbers and Fractions: Multiplying a whole number by a fraction:

   5 × 23 = 5 × 23 = 103 = 3135 \times \frac{2}{3} = \frac{5 \times 2}{3} = \frac{10}{3} = 3 \frac{1}{3}5 × 32 = 35 × 2 = 310 = 331

   The whole number 5 is multiplied by the fraction 23\frac{2}{3}32, and the result is an improper fraction, which can also be expressed as a mixed number.

Discussion:

The current integrated theory of numerical development distinguishes itself from approaches focused on privileged domains, evolutionary processes, and conceptual change by prioritizing the acquisition of knowledge regarding numerical magnitudes as a fundamental process that connects the understanding of all real numbers. This theoretical framework has generated several precise predictions that are not anticipated by the other theories. Notably, these alternative theories would not have foreseen that individual variations in knowledge of fraction magnitudes would strongly correlate with success in solving arithmetic problems involving fractions; that differences in knowledge of fraction magnitudes would be closely linked to individual variations in mathematics achievement test scores; or that performance across various tasks assessing fraction magnitude knowledge would show a high degree of correlation.

Limitations: Here are some limitations:

1. Constraints in cognitive development.
2. Variations in cultural and contextual approaches
3. Challenges of abstract thinking
4. Limitations in practical application
5. Influences of formal education structure

Conclusion:

The development of whole numbers and fractions can be viewed as a gradual transition from a fundamental grasp of counting numbers (whole numbers) to more intricate ideas involving segments of a whole (fractions). Whole numbers initially serve as the cornerstone for comprehending quantity, as well as the operations of addition and subtraction. As one progresses, fractions introduce the notion of division and the necessity for greater accuracy in depicting parts of a whole number.

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