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3-Minimally Nonouterplanar Total Block Graph in terms of forbidden subgraphs

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Abstract:

In this paper we establish characterizations of graphs whose 3-minimally Nonouterplanar total block graph, in terms of forbidden subgraphs.

Key words: Planar, Outerplanar, Nonouterplanar and minimally nonouterplanar.

INTRODUCTION

In [2] kulli introduced the concept of the semitotal-block graphs and total-blockgraps.In [3] and [4], the planarity and outerplanarity of these graph valued functions were discussed. In[5], one finds the minimally nonouterplanarity of these graph valued functions. In [1], D.G.Akka and M.S.Patil finds the 2-minimally nonouterplanarity of these graph valued functions. In [6], M.H. Muddebihal, jayashree.B.Shetty and Shabbir Ahmed finds the 3-minimally nonouterplanarity of these graphs valued functions. In this paper we obtain the characterizations of graph whose 3-minimally Nonouterplanar total block graph, in terms of forbidden subgraphs.

The folowing definations will be noted for later use. A graph G is called a block if it has more than one vertex, is connected and has no cutvertices block of a graph G is a maximal subgraph of G which itself a block.
If $B=\{u_1, u_2,\_ \_ \_ \_, u_r ; r\geq 2\}$ is a block of G, then we say that vertex u_1 and block B are incident with each other as are u_2 and B so on.
If two distinct blocks B_1 and B_2 are incident with a common cutvertex, than they are adjacent blocks. The vertices and blocks of a graph are called the members.

The following will be useful in the proof of our results.

Main Results:

Theorem 1:

Let G has a connected planar graph. Then it has a 3-minimally nonouter planar total block graph if and only if it has no subgraph to P_5K_1, K_4-x, K_4-x, H_1, H_2….,H_{15} or H_{16}.

Proof:

Let G be a connected planar graph with a 3-minimally nonouter planar total block graph. We now show that all graphs homemorphic to P_5K_1, K_4-x, K_4-x, H_1, H_2….,H_{15} or H_{16}, have not 3-minimally nonouter planar total block graph. It follows from Theorem B, since the graph homeomorphic to P_5+K_1 has four cycles as a block, the graph homeomorphic to K_4-x . K_4-x or H_3 has four cycles as two blocks, the graph homeomorphic to H_1 have two cut vertices of degree 3 and each lies on 3 blocks, graph homeomorphic to H_2 have a cut vertex of degree 3 and it lies on 3 blocks in which each block has no end vertex of G, the graph homeomorphic to H_4 has a cycle $C_n(n\geq8)$ together with a diagonal edge joining a pair of vertices of length at least 4, the graph homeomorphic to H_5 has a cycle $C_n(n\geq7)$ together with two diagonal edges each joining a pair of vertices of length two and three, the graph homeomorphic to H_6 to H_7 have to cycles C_4 as blocks and remaining blocks are edges, the graph homeomorphic to H_8 has a triangle together with three paths $P_n(n\geq3)$ incident at different vertices of a triangle, the graph homeomorphic to H_9 has a triangle together with three paths $P_n(n\geq2)$ incident at a vertex of a triangle, the graph homeomorphic to H_{10} has a cycle C4 together with an end edges adjoined at two alternate vertices of a cycle C_4, the graph homeomorphic to H_{11} has a cycle C_5 together with an end edges adjoined to two nonadjacent vertices of a cycle C_5, the graph homeomorphic to H_{12} has a cycle C_5 together with an two paths P_m and $P_n(m\geq2, n\geq2)$ adjoined at a vertex of a cycle C_5, the graph homeomorphic to H_{13} has three triangle such that it has a unique cut vertex which lies on three blocks, the graph homeomorphic to H_{14} has four triangles such that every cutvertex lies on three blocks and the graph homeomorphic to H_{16} has four triangles as blocks and remaining blocks are edges.

Conversely, assume that G contains no subgraphs homeomorphic to P_5+K_{19} K_4-x. K_4-x, H_1,H_2….,H_{15} or H_{16}.

We consider the following cases.

Case 1: Suppose G has a unique cutvertex v of degree v≥ 6. Then we consider the following subcases of case 1.

Sub Case 1.1 Assume Deg v = 6 and cutvertex v lies on 3 blocks of G in which each block is a triangle. Then G has a subgraph homeomorphic to H_{13}, a contradiction.

Subcase 1.2 Assume deg v = 5 and cutvertex v lies on 4 blocks of G. Such that one of these blocks contains a triangle and each of the remaining blocks is an edge. Then clearly G has a subgraph homeomorphic to H_9, a contradiction.

From the subcase 1.2 we conclude that G is a triangle together with two paths Pm ad Pn ($m \geq 2$, $n \geq 2$) incident at a same vertex of a triangle.

Sub Case 1.3 Assume degv =4 then we consider the following subcases of subcase 1.3.

Sub case 1.3.1. Suppose cut vertex v lies on 3 blocks of G, such that one of these blocks contains a cycle C_5 and each of the remaining block is an edge. Then G has a subgraph homeomorphic to H_{12}, a contradiction.

From the subcase 1.3.1 suppoe cut vertex v lies on 3 blocks of G_2 such that one of these blocks contains a cycle C_5 and each of the remaining block is an edge. Then G has a subgraph homeomorphic to H_{12}, a contradiction.

From the subcase 1.3.1. we conclude that G is a cycle C_5 together with a path Pn(≥ 2) incident to a vertex of a cycle C_5.

Subcase 1.3.2. Suppose cutvertex v lies on two blocks of G, such that each block contains a cycle C_4. Then G has a subgraph homeomorphic to H_6, a contradiction.

Subcase 1.3.3. Suppose cutvertex v lies on two blocks of G, such that each of these block contains K4-x. Then G has a subgraph homeomorphic to K_4-x.K_4-x, a contradiction.

Subcase 1.4 Assume deg v=3 and cutvertex v lies on 3 blocks of G in which each of these blocks has no end vertex of G. Then G has a subgraph homeomorphic to H_2, a contradiction.

Case 2: Suppose G has two cutvertices v_1 and v_2 of deg v_1 or deg $v_2 \geq 5$. Then we consider the following subcases of case 2.

Subcase 2.1. Assume deg v_1 = deg v_2 = 5 and both cutvertices v_1 and v_2 lies on 3 blocks each, such that each two blocks as triangles and each one of the remaining block is an edge. Then G has a subgraph homeomorphic to H_{15}, a contradiction.

Subcase 2.2. Assume $\deg v_1 = \deg v_2 = 3$. Then we consider the following subcases of subcases 2.2.

Subcase 2.2.1 Suppose cutvertices v_1 and v_2 lies on two blocks each, such that one of these block contains a cycle c_4 or c_5 and each of the remaining block is an edge. Then G has subgraph homeomorphic to H_{10} or H_{11}, a contradiction.

From the subcase 2.2.1 we conclude that G is a cycle C5 together with two paths Pm and Pn (m≥1, h≥1) adjoined at two consecutive vertices.

Sub case 2.2.2 Suppose each cutvertices v_1 and v_2 lies on two blocks each, such that each one of these block contains a cycle c4 and each of a remaining block is an edge. Then G has a subgraph homeomorphic to H_7, a contradiction.

Subcase 2.2.3: Suppose each cutvertices v_1 and v_2 lies on two blocks each, such that each one of these blocks contains a K_4-x and each of the remaining block is an edge. Then G has a subgraph homeomorphic to H3, a contradiction.

From the subcases 1.3.2, 1.3.3, 2.2.2 and 2.2.3 we conclude that G has exactly two cycles C3 and C4 as blocks, which are end blocks and every cut vertices of G lies on atmost two blocks.

Subcases 2.2.4 Suppose each cutvertices v_1 and v_2 lies on 3 blocks each, in which each block is an edge of G. Then G has a subgraph homeomorphic to H_1, a contradiction.

From the subcase 1.4 and subcase 2.2.4 we conclude that G does not contain a tree.

Case 3: Suppose G has three cut vertices as v_1, v_2 and v_3 of deg vi=3 (i=1,2,3).

Such that each of these cutvertices v_1, v_2 and v_3 lies on two blocks of G, then each one of these blocks contains a triangle and each of the remaining block is an edge. Then G hs a subgraph homeomorphic to H_8, a contradiction.

From the case 3 we conclude that G is a triangle together with paths Pm, Pn ($m \geq 2$, $n \geq 2$) and P_2 incident at different vertices.

Case 4 suppose G has atleast three diagonal edges exists in oen cycle $Cn(n \geq 6)$. Then G has a subgraph homeomorphic to P_5+k1, a contradiction.

Case 5. Suppose G has atleast two diagonal edges. Then there are two subcases to consider depending on whether the two diagonal edges exists in one cycle or in two different cycles.

Subcase 5.1. Assume two diagonal edges exists in one cycle of length two and three not from same verte to two consecutive vertices. Then G has subgraph homeomorphic to H_5, a contradiction.

Subccase 5.2 Assume two diagonal edges exists in different edge – disjoint cycles. Then G has a subgraph homeomorphic to K_4-x.K_4-x or H_3, a contradiction.

From each case 4, subcase 5.1 and subcase 5.2 we have a contradiction.

Hence G has exactly two diagonal edges a P_4+K_1 as each joining a pair of vertices of length exactly two or together with two diagonal edges each joining a pair of vertices of length two and three which are adjacent.

Case 6 Suppose G is a cycle $Cn(n \geq 8)$ together with a diagonal edge joining a pair of vertices of length atleast 4. Then G has a subgraph homeomorphic to H4, a contradiction. From the above case we conclude that G is a cycle $Cn(n \geq 6)$ together with a diagonal edge joining a pair of vertices of length triangle is of degree 3 or 4.

Then G has a subgraph homeomorphic to H_{14}, a contradiction.

Case 8: Suppose G has four triangles as blocks and remaining blocks are edges. Then G has a subgraph homeomorphic to H_{16}, a contradiction.

From the cases 7, 8 and sub case 2.1 and subcase 1.1 we conclude that G has exactly three triangles as blocks, such that every cutvertex of G lies on atmost two blocks and each triangles has atleast one vertex of degree two.

We have exhausted all possibilities. In each case we found that G has one of our forbidden subgraph P_5+K_1, K_4-x.K_4-x,H_1,H_2…H_{15} or H_{16}. Thus by theorem A, $i[T_B(G)]=3$.

This completes the proof.

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