International Journal For Advanced Research In Science & Technology
A peer reviewed international journal www.ijarst.in
ISSN: 2457-0362

THE LIE LAPLACIAN TRANSFORMATION AND ITS APPLICATIONS TO CERTAIN POLYNOMIALS AND OTHER SPECIAL FUNCTIONS

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ABSTRACT
This is the first time that generating functions are described in a broad manner. We may use the generating function to get the coefficients of a number of different polynomials. The n-th term of the polynomial is also calculated. Special functions, and more especially hypergeometric functions and polynomials in one or more variables, are frequently needed for addressing problems in physics, engineering, statistics, and operations research. In the theory of special functions, some authors have recently focused on generation functions, summations, and transformations formulas. Generating functions, finite sum characteristics, and transformations are crucial to the study of special functions. In view of the growing importance of generating functions, this dissertation covers many types of generating functions for special functions and polynomials in one, two, or more variables, such as linear, bilinear, bilateral, double, and multiple generating functions. These generating functions can be obtained by group theoretic techniques, the Nishimoto fractional calculus, integral operator methods, or the series rearrangement method.

Keywords: - Special Function, System, Sequences, Fractional, Function.

I. INTRODUCTION

It is common knowledge that generating functions play significant roles in the investigation of a broad variety of potentially useful properties and qualities of the sequences that they create. These functions are responsible for producing. The generating functions are responsible for the creation of these sequences. To convert differential equations describing discrete-time signals and systems into algebraic equations, generating functions are another tool that may be employed. Another use for creating functions is seen here. This makes it possible to simplify discrete-time system analysis as well as a wide variety of other problems that call for sequential fractional-order difference operators, operations research, and other areas of applied sciences. These advantages could also be applicable to a variety of other fields of applied research (including, for example, queuing theory and related stochastic processes). Utilizing generating functions in an efficient manner is what is meant by this technique $\{f_n\}_{n=0}^{\infty}$ by making the appropriate changes to Darboux's technique involves a number of steps, one of which is the examination of the asymptotic behaviour of the created sequence. This is an essential stage in the process.

II. GENERATING FUNCTIONS

In the next section of this essay, we are going to do our best to explain the overall shape of the generating function. In addition, we have the ability to extract the polynomial functions as well as their coefficients, both of which are essential components in the development of special functions.

$$G(x,t) = \sum_{n=0}^{\infty} F_n(x)t^n, (8)$$

In order to derive the $\mathrm{Fn}\ (\mathrm{x})$ using the equation, we get here

$$F_n(x) = \frac{1}{n!}\frac{\partial^n G(x,t)}{\partial t^n} \mid t = 0\ (10)$$

where the equation stands for the underlying framework of the method for aggregating the various polynomials as a whole. The next thing that has to be done is to find the coefficients of an, which are important in the special function. This may be found by following the instructions in the previous sentence. This is the next step that must be taken after the previous one. The revised version of the polynomial Fn(x) is presented below for your reference. It looks like this now:

$$F_n(x) = \sum_{n=0}^{\infty} a_n x^n\ (11)$$

Also we have

$$F_n(x) = \sum_{n=0} \frac{x^n}{n!}\frac{\partial^n F_n(x)}{\partial x^n}\ (12)$$

In conclusion, the equation gives us the opportunity to derive the following expression:

$$a_n = \frac{1}{n!^2}\frac{\partial^{2n} G_n}{\partial x^n\, \partial t^n}(x=0, t=0) = \frac{1}{n!}\frac{\partial^n F_n(x)}{\partial x^n}\ {}_{(x=0}$$

III. SOME POLYNOMIALS AND SPECIAL FUNCTIONS BY USING LIE LAPLACE TRANSFORMATION

When Courant and Hilbert were doing research in 1953 into the application of ordinary differential equations in the sphere of physics, they came into the concept of special functions by complete accident. As a direct consequence of this, the study of special functions eventually emerged as a distinct academic discipline. In addition, at the same time, Morse and Feshbach were looking at the ways in which special functions might be used to the research of difficulties that are related to the physical sciences. As a direct result of this, there have been developments made in the area of special functions as a consequence. On the basis of what he has seen, Paul Turan asserts that the history of special functions goes back a very, very, very long distance. This assertion is based on his observations. Euler, Legendre, Laplace, Gauss, Kummer, Riemann, and Ramanujan are just a few of the well-known mathematicians who were active in the 18th and 19th centuries and made significant contributions to the theory of special functions. Riemann and Kummer also played important roles in the development of the theory of special functions. The unique functions were the focus of study in the past, and for the same reasons they are the focus of research in the present day.

These include their application to a range of different subfields within physics and mathematics as well as their interaction with other subfields, such as number theory, combinatory, computer algebra, and representation theory. Other examples are also included. If a reader is interested in the subject matter, they should study the

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ISSN: 2457-0362
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exceptional books that have been written on the subject by various authors Andrews, Rao, Rose, and Rainville. Miller's work helped expand Weisner's theory by establishing a connection between that theory and the factorization technique developed by Schrodinger. This was Miller's contribution to the scientific community. Miller's contribution to the development of Weisner's theory consisted of the following. He then went on to demonstrate a link between the theory and the work done by Infield and Hull, which broadened the applicability of the theory even more. Kalnins, Onacha, and Miller have all contributed to study on the topic of Lie algebraic characterizations of two-variable Horn functions. This line of inquiry has been pursued by all three researchers. In order to do this, they stretch a two-variable Horn function into a set of hyper geometric functions with a single variable. Because of this, a technique for the development of generating functions is eventually conceived of and developed. The discussion of hyper geometric functions in one, two, and even more variables takes up the majority of the space allotted to this section of the thesis. Following a discussion of the definitions of basic functions like the Gamma and Beta functions as well as the significant elements of these functions, these functions are then presented as prospective solutions to the issue that is now being considered.

IV. GAUSSIAN HYPERGEOMETRIC FUNCTION

We will go over some definitions and identities using pochhammer's symbol (l), the Gamma function (r(z), and the related function in order to introduce the Gaussian hypergeometric series and its expansions. The purpose of this exercise is to introduce students to the Gaussian hypergeometric series and its many extensions.

The Gamma Function

The Gamma function is one of the most basic special functions, although it plays a very significant role (z). It may be defined in a number of different ways, the most of which can be traced back to Euler.

In creating the function, we are following Euler's lead. r(z) by

$$\Gamma(z)=\int_0^\infty t^{z-1}e^{-t}\,dt,\ \mathrm{Re}(z)>0$$

Partially integrating definition (1.2.1) produces the recurrence reiafion.

$$\Gamma(z+1)=z\,\Gamma(z)$$

From the relation and it follows that

$$\Gamma(z)=\begin{cases}\int_0^\infty t^{z-1}e^{-t}\,dt, & \mathrm{Re}(z)>0,\\ \dfrac{\Gamma(z+1)}{z}, & \mathrm{Re}(z)<0,\quad z\neq-1,-2,-3,\cdots\end{cases}$$

The practical answer is provided by the recurrence relation (1.2.2).

$$\Gamma(z+1)=z!\ ,\ z=0,1,2,\ldots\ldots$$

which proved the generation of the function z by the Gamma function Vz).

$$z!=\int_0^\infty t^{z}e^{-t}\,dt,\ z=0,1,2,\cdots\cdots$$

The Pochhammer's Symbol and the Factorial Function

The Pochhammer's symbol $(\lambda)_n$ is defined by

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ISSN: 2457-0362

$$(\lambda)_n = \begin{cases} 1, & if \ n=0 \\ \lambda(\lambda+1)(\lambda+2)...(\lambda+n-1) & , if \ n=1,2,3,.... \end{cases}$$

Since $(1)_n = n!$, $(\lambda)_n$ may be thought of as an extension of the simple factorial, which is represented by the symbol $(\lambda)_n$ is equivalent to the factorial operation in other contexts. The Gamma function is defined as

$$(\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)}, \lambda \neq 0,-1,-2, \cdots\cdots$$

In addition to this, the coefficient of the binomial distribution may now be stated as

$$\binom{\lambda}{n} = \frac{\lambda(\lambda-1)\cdots\cdots(\lambda-n+1)}{n!} = \frac{(-1)^n(-\lambda)_n}{n!}$$

or, equivalently, as

$$\binom{\lambda}{n} = \frac{\Gamma(\lambda+1)}{n!\Gamma(\lambda-n+1)}.$$

It follows from (1.2.8) and (1.2.9) tha

$$\frac{\Gamma(\lambda+1)}{\Gamma(\lambda-n+1)} = (-1)^n \ (-\lambda)_n,$$

which, for $\lambda = \alpha - 1$, yields

$$\frac{\Gamma(\alpha-n)}{\Gamma(\alpha)} = \frac{(-1)^n}{(1-\alpha)_n}, \alpha \neq 0, \pm1, \pm2, \cdots\cdots.$$

Equations (1.2.7) and (1.2.11) suggest that

$$(\lambda)_{-n} = \frac{(-1)^n}{(1-\lambda)_n}, n=1,2,3,\cdots\cdots; \lambda \neq 0, \pm1, \pm2, \cdots$$

And

$$(\lambda)_{m+n} = (\lambda)_m \ (\lambda+m)_n,$$

which in conjunction with (1.2.12), gives

$$(\lambda)_{n-k} = \frac{(-1)^k(\lambda)_n}{(1-\lambda-n)_k}, 0 \leq k \leq n$$

For $\lambda = 1$, we have

$$(n-k)! = \frac{(-1)^k n!}{(-n)_k}, \ 0 \leq k \leq n,$$

which may be written as:

$$(-n)_k = \begin{cases} \frac{(-1)^k n!}{(n-k)!}, & 0 \leq k \leq n, \\ 0, & k > n. \end{cases}$$

V. CONCLUSION

Many novel functions have been developed within the framework of differential equations theory. Specialized operations are what we refer to here. The theory of special functions is fundamental to the formalisation of mathematical physics and applied mathematics. First established by Euler, Gauss, Laplace, Bessel, Legendre, Jacobi, Hermit, Laguerre, and others, then expanded upon by Whittaker, Watson, Ramanujan, Appell, Ragab, Hardy, Lebdev,Erdelyi, Chaundy, Bailey, and many others, and finally, continually refined by new achievements and suggestions within the context of applied sciences.

In the fields of physics, engineering, statistics, and operations research, special functions, in general, and hypergeometric functions and polynomials in one or more variables, in particular, come up rather often in a broad range of issues. During the course of the last few years, a number of writers will have paid some attention to various aspects of theory pertaining to special functions, including generation functions, summations, and transformations formula. When it comes to the study of special functions, generating functions, finite sum characteristics, and transformations are all very important aspects. In light of the increasing significance of generating functions, this synopsis will include specific classes of generating functions. These classes include linear, bilinear, bilateral, double, and multiple generating functions for specific special functions and polynomials with one, two, or multiple variables. The series rearrangement approach, integral operator methods, Nishimoto's fractional calculus, and the group-theoretic method are the techniques that are used to produce such generating functions. Additionally, several transformations, fractional derivative formulae, and finite sum characteristics for specific hypergeometric functions and polynomials are provided, and a variety of special instances are derived from these qualities.

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